INTRODUCTION TO BIOSTATISTICS

How to read the standard normal table

(Table A.3 of Principles of Biostatistics, Pagano, M. and Gauvreau, K.)

When reading a normal table, we take advantage of the following features of the normal distribution:

- The **symmetry** of the standard normal curve around zero (its mean). Thus, \( P(Z \geq z) = P(Z \leq -z) \), where \( z \geq 0 \).
- The fact that (as in any distribution) the area under the curve is equal to 1. Thus, \( \{Z \geq z\} \) and \( \{Z \geq z\} \) are two complementary events, \( P(Z \geq z) = 1 - P(Z \leq z) \).

We are usually faced with two problems:

1. Given a number \( z \geq 0^1 \) (say) find \( p \) such that the following is true:
   
   1.1. \( P(Z \geq z) = p \) Read \( p \) directly from standard normal table\(^2\).
   1.2 \( P(Z \leq -z) = p \) Read \( p_1 = P(Z \geq z) \) from the normal table
      \( p = p_1 \) (by the symmetry of the normal distribution).
   1.3 \( P(Z \leq z) = p \) Read \( p_1 = P(Z \geq z) \) from the normal table: \( p = 1 - p_1 \).
      Notice that \( \{Z \geq z\} \) and \( \{Z \leq z\} \) are complementary events.
   1.4 \( P(Z \geq -z) = p \) Read \( p_1 = P(Z \geq z) \) from the normal table: \( p = 1 - p_1 \).

   Now assume that \( z_1 \leq z_2 \):
   
   1.5 \( P(z_1 \leq Z \leq z_2) = p \) Calculate \( p_1 = P(Z \geq z_1) \) (if \( z_1 \geq 0 \) refer to 1.1, if \( z_1 < 0 \) refer to 1.4) and \( p_2 = P(Z \geq z_2) \) (if \( z_2 \geq 0 \) refer to 1.1,
      if \( z_2 < 0 \) refer to 1.4); then \( p = p_1 - p_2 \)

   Special case: \( z > 0 \)
   
   \( P(-z \leq Z \leq z) \) Read \( p_1 = P(Z \geq z) \) from the normal table \( p = 1 - 2p_1 \).
   Notice that this is the **central** part of the distribution.

2. Given a probability \( p \) find \( z \) such that the following is true:

   2.1 \( P(Z \geq z) = p \)
      
      If \( p \leq 0.5 \) Then \( z \geq 0 \): Look up \( p \) in table. \( z \) is the closest number\(^3\).
      
      If \( p \geq 0.5 \) Then \( z \leq 0 \): Look up \( p_1 = 1 - p \) in the table. Locate the closest number. \( z \) is its negative.

   2.2 \( P(Z \leq z) = p \)
      
      If \( p \leq 0.5 \) Then \( z \leq 0 \): Look up \( p \) in table. Locate the closest number. \( z \) is its negative.
      
      If \( p \geq 0.5 \) Then \( z \geq 0 \): Look up \( p_1 = 1 - p \) in the table. \( z \) is the closest number.

   2.3 \( P(-z \leq Z \leq z) = p \) Look up \( p_1 = (1 - p)/2 \) in the table. \( z \) is the closest number. \( -z \) is its negative.

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\(^1\) Recall that capital \( Z \) is the (normally distributed) random variable, while \( z \) is the values it assumes.

\(^2\) The \( p \) corresponding to \( z \) is read by going “down” in the table as many lines as it’s necessary to approach \( z \) as closely as possible (without going over), and then going “across” on the same line, as many columns as it’s necessary to approach \( z \) as closely as possible (without going over).

\(^3\) The “closest number” \( z \) corresponding to a given \( p \) is found by adding the number in the left margin of the line where \( p \) is located in the table, to the number at the top margin of the column where \( p \) is located in the table.