Decisions, Games, and Negotiation

Today’s Agenda

• Introduction to Game Theory
• Two-person matrix games
Where this fits in

Individual  Group

Individuals act separately

Game Theory  Negotiations
Why study game theory

• Parlor games (especially poker)
• Monopolies and duopolies
• Competitive pricing
• Military strategy
• Auctions
• Nuclear war
Perspectives

• Descriptive: how people actually behave
• Normative: what game theorists think is optimal
• Prescriptive: how you should behave, given that not everyone is perfectly rational
Setting the stage

- You have a set of choices
- You do not know what your opponent will choose
- Your payoff will depend on what you choose and what your opponent chooses
- You cannot make binding commitments
Simplest example: bi-matrix game

- Two parties
- Parties choose simultaneously between two alternatives
- Perfect information
  - You know payoffs for both parties
  - Even stronger: you know that your opponent knows that you know that… (common knowledge)
- The game always stays the same
Bi-matrix games

• Two parties
  – Row player A
  – Column player B
• A’s alternatives (his strategy set)
  – U for Up and D for Down
• B’s strategy set
  – L for Left and R for Right
• Payoffs are what each person receives if a certain pair of strategies is chosen.
Bi-matrix games

Payoffs: $A$ receives $x$ and $B$ receives $y$ if $A$ plays $U$ and $B$ plays $L$

$u_A(U,L) = x$ and $u_B(U,L) = y$
Bi-matrix games

Denardo writes this as two matrices

\[ A = \begin{pmatrix} x \\ x \end{pmatrix} \quad B = \begin{pmatrix} y \\ y \end{pmatrix} \]
Zero-sum Games

Special case of bi-matrix games

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>x, -x</td>
<td>y, -y</td>
</tr>
<tr>
<td>D</td>
<td>z, -z</td>
<td>w, -w</td>
</tr>
</tbody>
</table>

Payoff for one player is always the opposite of the payoff for the other player.
Example 1

Player B

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>4,3</td>
<td>3,0</td>
</tr>
<tr>
<td>D</td>
<td>12,8</td>
<td>5,4</td>
</tr>
</tbody>
</table>
Example 1

Player A

U
4, 3
12, 8

Player B

L
3, 0
5, 4

D
Example 1

Player A

Player B

<table>
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<tr>
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<tbody>
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Dominance

• No matter what B does, choice D is better than choice U for A.
• So player A will always pick D.
• No matter what A does, choice L is better than choice R for B.
• So player B will always pick L.
• Equilibrium strategy is (D,L).
Definition of dominance

• Strategy s dominates strategy r for player A if s does better than r regardless of what strategy player B plays.

• More formally, the following inequality must hold for all strategies t in player B’s strategy set (if the inequality is strict, we say that s strictly dominates r):

\[ u_A(s,t) > u_A(r,t) \]
### Example 2

#### Player B

<table>
<thead>
<tr>
<th></th>
<th>L</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>U</strong></td>
<td>0, 2</td>
<td>5, 4</td>
</tr>
<tr>
<td><strong>D</strong></td>
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Example 2

Player B

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</tr>
</tbody>
</table>

Player A
Example 2

Player A

Player B

L    R

U

0,2  \hspace{2cm} 5,4

D

10,3 \hspace{2cm} 3,8
Dominance revisited

• Player A does not have a dominant strategy at first.
• But, R dominates L for player B.
• A knows that B will never play L.
• Choice U dominates D for player A given that L has been ruled out.
• Equilibrium strategy is (U,R).
• “Iterated elimination of (strictly) dominated strategies.”
Dominance revisited

• A rational player will never play a dominated strategy. (Why?)
• The other player knows this and will adjust her play accordingly.
• The first player knows that the other player will do this and adjusts his player accordingly and so on.
• Upshot: we have to pay attention to the thought processes of other players.
Example 3

<table>
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<tr>
<th></th>
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<tbody>
<tr>
<td>U</td>
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<td>8, 6</td>
</tr>
<tr>
<td>D</td>
<td>6, 8</td>
<td>10, 4</td>
</tr>
</tbody>
</table>
Example 3

Player B

<table>
<thead>
<tr>
<th></th>
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<th>R</th>
</tr>
</thead>
<tbody>
<tr>
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Player A
Example 3

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Player A

Player B
Flaws in dominance?

• Iterated elimination of dominated strategies suggests that (D,L) is the dominant strategy.
• But how much do you trust A to be perfectly rational?
• That -100 payoff is awfully scary…
Example 4

Player B

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<tbody>
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<tr>
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</tbody>
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Player A
A bigger flaw with dominance

• A game is dominance-solvable if iterated elimination of strictly dominated strategies produces a single prescription for rational play.

• Not all games are dominance-solvable!

• Need a weaker solution concept.
Best-response

• A strategy $s$ in player A’s strategy set $S$ is called a *best-response* to player B choosing strategy $t$ in her strategy set $T$ if the following inequality is true for all other strategies $s’$ in $S$

$$u_A(s,t) \geq u_A(s’,t)$$

We denote this $\text{BR}_A(t) = s$
Equilibrium

- The strategy pair \((s, t)\) is a Nash equilibrium if \(s\) is player A’s best-response to player B choosing \(t\), and also \(t\) is B’s best-response to A choosing \(s\):

\[
\text{BR}_A(t) = s \quad \text{and} \quad \text{BR}_B(s) = t
\]
Example 4

Player B

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<tr>
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<tbody>
<tr>
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Player A
Equilibrium in Example 4

• (U,R) and (D,L) are both Nash equilibrium strategy pairs
  – U is a best response to B playing R and R is a best response to A playing U.
  – D is a best response to B playing L and L is a best response to A playing D.

• Are there any others?
More on equilibrium

- Can be more than one Nash equilibrium
- A dominated strategy will never be played in a Nash equilibrium. (Why?)
- Stronger notion than equilibrium is stability: no group of participants can benefit by changing their strategies.
- Important question: is there always a Nash equilibrium?
Nash existence theorem

• Special case of the Nash existence theorem applies to bi-matrix games.
• All bi-matrix games have at least one Nash equilibrium in mixed strategies.
• Mixed strategies are a probability distribution over pure strategies (we will discuss this more next lecture).
• See Denardo 16.8 for more information.
Example 5

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>C</th>
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</thead>
<tbody>
<tr>
<td>D</td>
<td>-5, -5</td>
<td>0, -10</td>
</tr>
<tr>
<td>C</td>
<td>-10, 0</td>
<td>-2, -2</td>
</tr>
</tbody>
</table>
Example 5

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>-5, -5</td>
<td>0, -10</td>
</tr>
<tr>
<td>C</td>
<td>-10, 0</td>
<td>-2, -2</td>
</tr>
</tbody>
</table>
Example 5

Player B

Player A

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>-5 , -5</td>
<td>0 , -10</td>
</tr>
<tr>
<td>C</td>
<td>-10 , 0</td>
<td>-2 , -2</td>
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</table>
Social Dilemma

- (D,D) is the dominant strategy, but (C,C) is obviously better for both parties
- Perfectly rational people do worse off than naïve, trusting people
- What if we repeated the game 20 times and kept track of the cumulative payoffs?

- What’s your first move?
Social Dilemma

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>D</td>
<td>D</td>
<td>C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| B | C | D | D |     |    |?
Some insights

• Trust is hard to acquire and easy to lose
• You don’t need to actually trust your opponent
• Start by being nice, but…
• …respond to defections by your opponent…
• …without being vindictive
  – Tit for tat
• Simple strategies beat complex ones
• Beware of the last few plays of the game
Interpretations

- Tragedy of the commons
- Global warming
- Public goods
- Charitable giving
- Free-riders