Decisions, Games, and Negotiation

TODAY’S AGENDA:
Review Questions and Consolidation

• Especially net present value and linearity.
Decisions, Games, and Negotiation

Review Questions Received

- Dimensions of linear programming
- Trees
- Trade offs among objectives
- Shadow prices
- Sourcebooks
- Net present value, interest rates.
Decisions

• Want to rank alternatives in order.

• Numbers have a familiar ordering.

• Value functions express preference ordering using numbers: \( v(A) > v(B) \) if and only if alternative A is preferred to alternative B.
Value Functions

• The objective function in a LP is an example of a value function because it takes alternatives and assigns numbers according to how desirable each is.

• And, in LP, it does this in a linear way.
Alternatives and their Attributes

• In LP, each alternative is determined by a list of attributes.

• Each attribute is measured by numbers, but the units for different attributes are not necessarily the same, e.g.

• A = ($50, 100 euro, 25 yen)
• A= (250 lbs, 2 tons, 10 kilos)
Proportionality and Additivity

- $A = ($50, 100\ euro, 25\ yen) \quad v=\text{profit}$
- $A = (250\ lbs, 2\ tons, 10\ kilos) \quad v=\text{weight}$

- Contribution of each alternative to objective function is \textit{proportional} to its level.

- Contributions of various attributes add together to get the total value of an alternative.

- Hence $v$ is linear! Coefficients convert to units to a common “numeraire” unit of value.
Proportionality and Additivity

• Contribution of each alternative to objective function is proportional to its level. (More realistic if amounts don’t vary by too much).

• Contributions of various attributes add together to get the total value of an alternative. (Depends on situation. There may not be a common “numeraire” unit to facilitate intercomparability of attributes.)
Additivity?

1. Suppose that you are having a two-course lunch where the price is fixed regardless of what you choose. The entrees are either Lasagna or Chef's Salad. The desserts are either Fruit Salad or Cheese Cake. Rate each of these four individual dishes on a scale of 1 to 10 where 10 is the most preferable to you.

- Lasagna __________
- Chef's Salad __________
- Cheese Cake __________
- Fruit Salad __________
9

Additivity?

2. Now rate each of your four possible meals by adding up the points you have assigned each dish.

• Lasagna and Cheese Cake

• Chef’s Salad and Cheese Cake

• Chef’s Salad and Fruit Salad

• Lasagna and Fruit Salad
Additivity?

2. Now rate each of your four possible meals by adding up the points you have assigned each dish.

- Lasagna and Cheese Cake __________
- Chef's Salad and Cheese Cake __________
- Chef's Salad and Fruit Salad __________
- Lasagna and Fruit Salad __________

- Does the ranking of meals by totaling the dish points reflect your true lunch preferences?
Discounting

$ next yr

$ now
Discounting

iso-NPV line:
$x + d\ y = \text{const}$
Discounting

iso-NPV line:
\[ x + d \ y = \text{const} \]
slope is \(-(1+r)\).

NPV is \(x\) intercept.

$ next yr

$ now
Net Present Value

NPV of an alternative with cash flow \( x_i \) in the \( i \)th year:
\[
v(A) = x_0 + d x_1 + d^2 x_2 + d^3 x_3 + d^4 x_4 + \ldots
\]

This is a normative conclusion: follows rationally from assumptions about independence, stationarity.

Behaviorially, experimenters also observe “hyperbolic discounting”: apple today vs. two tomorrow? One in a year vs. two in a year and a day?
Net Present Value

Imagine an alternative with cash flow \( x_i \) in the \( i \)th year:

\[ A = (x_0, x_1, x_2, x_3, x_4, \ldots) \]

Then Net Present Value of this alternative is:

\[ v(A) = x_0 + d x_1 + d^2 x_2 + d^3 x_3 + d^4 x_4 + \ldots \]

Where \( d = 1 / (1+r) \) is the discount rate that converts the units “\$ a year from now” into “\$ now.”
What is the NPV of an unending stream of $1 payments, starting now, discounted at rate 
\[ d = \frac{1}{1+r} \] ?

\[ S = 1 + d + d^2 + d^3 + \ldots \]
What is the NPV of an unending stream of $1 payments, starting now, discounted at rate $d = 1/(1+r) < 1$?

\[ S = 1 + d + d^2 + d^3 + \ldots \quad (1) \]

\[ dS = d + d^2 + d^3 + \ldots \quad (2) \]

\[ S - dS = 1 \quad (3) \]

\[ S = 1/(1 - d) \] and since $d = 1/(1 + r)$,

\[ S = (1+r)/r \quad (4) \]