Mathematics E-126:

Decisions, Games, and Negotiation

Assignment 8: Due Wednesday 4/20/05 by 5 pm EST.
Please review the handout on Assignment Policies for information on how to submit your assignment.

A. Reading
Review all the reading, slides, and homework for the course so far. Take a look at the “Challenging Review Problems” at the end of this assignment. You are not expected to turn those problems in and problem 4 in particular is much more challenging than will appear on the exam.

B. Required
Write and solve a problem that would be appropriate for the final exam. Particularly good ones might just show up on the test. A few examples of challenging review questions are included on the attached page to provide inspiration. (30 points)

Original problem (15 points)
Correct solution (15 points)

C. Choice
Define 10 of the following terms (3 points each). For each term you define, give an example to illustrate your point (2 points each). (50 points)

Linear programming, objective function, shadow price, sensitivity analysis, slack constraint, binding constraint, perturbation theorem, corner principle, break-even analysis, Borda count, even swaps, dominance, linearity, transitivity, money pump, =SUMPRODUCT, expected monetary value, risk aversion, risk premium, PrOACT, classical, frequentist, subjectivist, canonical probability, subjective probability, expectation, variance, binomial distribution, marginal probability, conditional probability, joint probability, Bayes’ theorem, Dutch book, flipping a tree, five column method, statistical independence, value function, utility function, basic reference lottery ticket, expected value of perfect information, expected value of sample information.

Here is how one excellent student response from last year looked like:

Certainty Equivalent: For a given decision maker (DM), the certainty equivalent of a lottery is the monetary quantity such that the DM is totally indifferent between receiving that amount for certain or taking a chance on the lottery. The certainty equivalent is affected by the shape of the DM’s utility function or, alternately, how his or her risk profile looks like. For any venture, the utility of the certainty equivalent equals the expected utility value of that venture. For a risk-neutral DM, the certainty
equivalent of a lottery is the same as the lottery’s EMV; for a risk-averse DM, the certainty equivalent is somewhat smaller. Example: because of the declining marginal utility of money, we find that wealthy people are generally less risk-averse when it comes to wagering small amounts. Thus, we would expect to see that a rich person’s certainty equivalent of a given lottery is larger than a poor person’s certainty equivalent of that same lottery.

D. Writing

Consider the following game you have been invited to play by an acquaintance who always pays her debts. Your acquaintance will flip a fair coin. If it comes up heads, you win $2. If it comes up tails, she flips again. If heads occurs on the second toss, you win $4. If tails, she flips again. If heads occurs on the third toss, you win $8, and if tails, she flips again, and so on. (20 points)

(a) What is the expected monetary value of this game?

<table>
<thead>
<tr>
<th>N</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>…</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>1/2</td>
<td>1/4</td>
<td>1/8</td>
<td>1/16</td>
<td>1/32</td>
<td>…</td>
<td>1/(2^K)</td>
<td></td>
</tr>
<tr>
<td>Payoff</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
<td>…</td>
<td>2^K</td>
</tr>
</tbody>
</table>

\[ \text{EMV} = \sum_{k=0}^{\infty} \frac{1}{2^k} \cdot 2^k \]

\[ = \sum_{k=0}^{\infty} 1 \]

\[ = \infty \]

(b) What is the most you personally would pay to play this game?

In all likelihood, only a finite amount

(c) Would you be indifferent between playing the game and receiving the expected monetary value for sure?

Almost certainly yes (an infinite payoff for sure? Of course!).

(d) What is so paradoxical about the St. Petersburg paradox and what concepts from the course can you use to resolve it?

The key concept is the declining marginal utility of wealth (a.k.a. concavity of the utility function or risk-aversion). We are not indifferent between a 1% shot at $1,000,000 and receiving $10,000 for certain, most likely preferring the latter.
E.g. suppose that your utility function were log_2(Wealth). Then, assuming that you started with zero wealth, we have:

\[ N = \text{consecutive times the coin comes up tails} \]

<table>
<thead>
<tr>
<th>N</th>
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</tr>
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</table>

| Payoff | 2  | 4  | 8  | 16 | 32 | 64 | … | 2^K |
| Utility | 1  | 2  | 3  | 4  | 5  | 6  | … | K+1 |

\[ \text{EU} = \sum_{k=0}^{\infty} \frac{1}{2^k} \cdot (k + 1) \]

\[ = 4 \]

Certainty equivalent = \( U^{-1}(4) = 2^4 = 16 \)

So under these assumptions, you would be willing to pay at most $16 to play the game.

**E. Preparation**

Professor Emeritus Howard Raiffa will begin lecturing next week with his perspectives on the course so far. Please read Chapter 16 of *Decision Making* and Chapters 4 and 5 of *Negotiation Analysis* in advance.