Decisions, Games, and Negotiation

Assignment 9: Due Wednesday 4/27/04 by 5 pm EST.
Please review the handout on Assignment Policies for information on how to submit your assignment.

A. Reading
Make sure you have read Chapter 16 of Decision Making and Chapters 4 and 5 of Negotiation Analysis before working on this assignment.

B. Required
Games and Linear Programming. (60 points)

Each of two players simultaneously reveals one of three words: stone, paper, or scissors. If their words are the same, the game is a draw. Otherwise, one player pays $1 to the other according to the following convention: scissors defeats (cuts) paper, paper defeats (covers) stone, and stone defeats (breaks) scissors.

1. Write down the 3 by 3 payoff matrix for this zero-sum game. Show that there is no equilibrium in pure strategies (hint: one way that you can do this is to look at best responses and show that there are always profitable deviations, but there is an easier way that works for zero-sum games).

2. Set up a linear program that can compute the mixed-strategy equilibrium using Excel.

Rowena adopts a mixed strategy (p1, p2, p3) where p1 is the probability she chooses stone, p2 is the probability she chooses paper, and p3 is the probability she chooses scissors.

Rowena’s expected payoff is then p2-p3 if Colin plays stone, p3-p1 when Colin plays paper and p1-p2 when Colin plays scissors. Colin wants to keep Rowena from achieving more than the minimum of these three values, namely, \( v = \min (p2-p3, p3-p1, p3-p1) \). Rowena, on the other hand, should try to make the quantity \( v \) as large as possible by adjusting her mixed strategies. She can do this by solving the linear program that maximizes the variable \( v \) subject to the constraints:

\[
\begin{align*}
v &\leq p2-p3 \\
v &\leq p3-p1 \\
v &\leq p1-p2
\end{align*}
\]
along with the constraints that her mixed strategies are actually probabilities. Code this up in an Excel worksheet that has a place for the entire 3 by 3 matrix for this game.

3. If Colin plays mixed strategies \((q_1, q_2, q_3)\), what linear program would he want to solve? Find his best strategy. Compare this strategy to the one

4. Now that you have Excel set up to deal with this particular 3 by 3 game, make up another 9 numbers that describe outcomes for another zero-sum game where each player has three options to choose from, and then compute the equilibrium using Solver.

C. Choices  
Do either problem 13 or 16 in §16.10 of Decision Making. (25 points)

D. Writing  
Write a paragraph explaining and illustrating one type of game that we have learned about. For examples of different games, please review this week’s readings and lectures. Discuss a current event that can be modeled with this game. Does your analysis shed any light on the problem or possible solutions? (15 points)

E. Preparation  
Plan to regularly use the discussion forum. Remember that participation in the discussion forum is part of your grade.

1. Suppose that you have devised a mixed strategy for a zero-sum game. In practice, would you really play the strategy that an urn draw or some other canonical probability device told you to play? How could you defend your decision if the outcome turns out not so good for you?

2. In the novel Catch-22, Joseph Heller describes a character who is asked, “What if everyone did that?” and replies, “Then I would be a damn fool not to.” Discuss with reference to the Prisoners’ Dilemma and with real life examples.