Mathematics E-126:

Decisions, Games, and Negotiation

Assignment 1: Due Wednesday 2/16/05 at 5 pm EST. 
Please review the handout on Assignment Policies for information on how to submit your assignment.

A: Reading  
Read Chapter 2 of *The Science of Decision Making* before coming to class or working on this assignment. Think about the following questions as you read. (You do not have to hand in your answers.)

1. What are the ingredients in a linear programming problem? What does it take to specify such a problem? When are two of them the same?


B: Required  
1. Consider the following functions of the variables x, y, and z:

   (a) $7x + 3y - 25z$
   (b) $5x$
   (c) $7x + 4y^2 + 7$
   (d) $x + xy + 13y - 8z$

   Which functions are linear? Which functions are nonlinear? Write down one more linear function and one more nonlinear function. Which functions could serve as valid objective functions for a linear program? Explain. **(20 points)**

2. Do Problem 7 on page 45 of *The Science of Decision Making*. The problem is reproduced below. You will need to use Microsoft Excel and Solver. **(35 points)**

   **(A Woodworking Shop)** A woodworking shop makes two products, cabinets and tables. Each cabinet sells for $3100, and each table sells for $2100. The raw materials that are needed to produce each cabinet and table cost $1200 and $700, respectively. The variable labor costs needed to make each cabinet and table are $1100 and $900, respectively. The company’s carpentry shop has a capacity of 120 hours per week. Its finishing shop has a capacity of 80 hours per week. Making each cabinet requires 20 hours of carpentry and 15 hours of finishing. Making each
table requires 10 hours of carpentry and 10 hours of finishing. The company wishes to determine the product mix that maximizes profit.

(a) Create a table showing all relevant data (c.f. Table 2.1 on page 19).
(b) Write down a linear program whose optimal solution maximizes profit.
(c) Use Solver to find this optimal solution. Explain why this is the solution.
(d) A way has been found to reduce the time needed to finish each cabinet from 15 hours to 12. Reformulate the linear program. Solve it again. Does the optimal solution change? If so, is there anything surprising about the way in which it changes?

C: Choices

Read problems 11, 13, or 14 in Chapter 3 of The Science of Decision Making (reproduced below from pages 90-92) and choose one that looks interesting. You do not need to solve the problem you select. Instead, just answer questions (a) through (f) below concerning the set up of the one problem you wish to start studying. (30 points)

11. (Subway Cars) The Transit Authority must repair 100 subway cars per month, and it must refurnish 50 subway cars per month. Each task can be done in its own facility. Each task can also be contracted to private shops but at a higher cost. Private contracting increases the cost by $2000 per car repaired and $2500 per car refurnished.

The Transit Authority repairs and refurnishes subway cars in four shops. Repairing each car consumes $\frac{1}{150}$ of the monthly capacity of its Evaluation shop, $\frac{1}{60}$ of the monthly capacity of its Assembly shop, none of the monthly capacity of its Paint shop, and $\frac{1}{60}$ of the monthly capacity of its Machine shop. Refurnishing each car consumes $\frac{1}{100}$ of the monthly capacity of its Evaluation shop, $\frac{1}{120}$ of the capacity of its Assembly shop, $\frac{1}{40}$ of the capacity of its Paint shop, and none the monthly capacity of its Machine shop.

13. (A Farmer) A 1200-acre farm includes a well that has a capacity of 2000 acre-feet of water per year. (One acre-foot is one acre covered to a depth of one foot). The farm can be used to raise wheat, alfalfa, and beef. Wheat can be sold at $550 per ton and beef at $1300 per ton. Alfalfa can be bought or sold at the market price of $220 per ton. Each ton of wheat that the farmer produces requires one acre of land, $50 of labor, and 1.5 acre-feet of water. Each ton of alfalfa that she produces requires $\frac{1}{3}$ acre of land, $40 of labor, and 0.6 acre-feet of water. Each ton of beef she produces requires 0.8 acres of land, $50 of labor, 2 acre-feet of water, and 2.5 tons of alfalfa. She can neither buy nor sell water. She wishes to operate her farm in a way that maximizes its annual profit.
14. **(Pollution Control)** A company makes two products in a single plant. It runs this plant for 100 hours each week. Each unit of product A that the company produces consumes two hours of plant capacity, earns the company a contribution of $1000, and causes, as an undesirable side effect, the emission of 4 ounces of particulates. Each unit of product B that the company produces consumes one hour of plant capacity, earns the company a contribution of $2000, and causes, as undesirable side effects, the emission of 3 ounces of particulates and 1 ounce of chemicals. The EPA (Environmental Protection Agency) requires the company to limit particulate emission to at most 240 ounces per week and chemical emissions to at most 60 ounces per week.

(a) Write down letters to represent the decision variables in the problem.
(b) Write one sentence describing the decision maker’s objective.
(c) Now, write out the objective function.
(d) Write one sentence describing each constraint.
(e) Now, write out equations that represent each constraint.
(f) Without calculating, make and explain your guess at the optimum.

**D: Sentences**

Write your own “help entry” that would explain to a new Excel user what the SUMPRODUCT function does, as well as how and when to use it. Include illustrations and list some possible applications. **(15 points)**

**E: Preparations**

For next week, read Chapter 3 of *The Science of Decision Making*. Glance at Chapter 4, too. Think about the following topics. (You do not have to hand in any of your answers.)

1. Under what circumstances does a linear program approximate reality well? When does it not?
2. How is the shadow price related to opportunity cost?
3. How are reduced cost and marginal cost related to opportunity cost?
4. Explain what “break even analysis” is all about, with examples.
5. What is the optimal solution to a linear program when the slope of the objective function is parallel to a side of the convex feasible region? Consider the feasible region graphed in Figure 3.5 on page 73 of *The Science of Decision Making*. 